ASSIGNMENT SET – I

Mathematics: Semester-III

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-302

Paper: Transforms and Integral Equations

Answer all the questions

1. Who first coined the term Wavelet? Define 'mother wavelet' and explain the utility of it.
2. Define eigen value and eigen function of an integral equation.
3. Define inverse Fourier transform with the conditions of existence of the transform.
4. When a function $f(x)$ is said to be of exponential order$O(e^{ax})$ for x>0? If $f(x)$ is of exponential order what extra condition is needed for the existence of its Laplace transform?
5. Prove that the convolution operator of Laplace transform is commutative.
6. Verify the initial value theorem in connection with Laplace transform for the function $2 + cos(t)$.
7. Define the Wavelet function and analyze the parameters involving in it.
8. Define Heaviside step function and then find its Laplace transform.

   using Laplace transform find the solution of the equation $\frac{d^2y}{dx^2} + 4y = u(x-2)$ where'u' is the unit step function satisfying the boundary conditions $y(0) = 0, y'(0) = 1$.

9. Prove that the Fourier transform of $\frac{1}{x}$ is i$\sqrt{\frac{\pi}{2}}$ sgn(x), where sgn(x) is signum function.
10. State and prove Parseval's identity on Fourier transform.
11. With help of the resolvent kernel, find the solution of the integral equation

$$y(x) = 1 + x^2 + \int_0^x (\frac{1+x^2}{1+t^2}) y(t)dt.$$

12. Find the solution of the following problem of free vibration of a stretched string of infinite length PDE: $\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ =0, $-\infty < x < \infty$, with the boundary conditions $u(x,0) = f(x)$, $-\infty < x < \infty$,$\frac{\partial u(x,o)}{\partial t}$ =$g(x)$, u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$.
13. Find the value of $sin(t) * t^2$ where $*$ denotes the convolution operator on Laplace transform.
14. Find the Fourier Transform of Dirac-delta function.
Reduce the boundary value problem
15. $\frac{d^2y}{dx^2}$+$\lambda y = 0$, with boundary condition $y(0) = 0, y(1) = 0$ to an integral equation and find its kernel.
16. Find the Laplace transformation of $f(x) = [x]$ where $[x]$ is the greatest positive integer less than or equal to x.
17. Solve the integral equation by Laplace equation by Laplace transformation

$$\frac{dy}{dt} + 5\int_0^t cos2(t-u)y(u)du = 10 \text{ where } y(0) = 2$$

18. Using the method of Fourier transform determine the displacement $y(x, t)$ of an infinite string, given that the string is initially at rest and that the initial displacement is $f(x)$, ($-\infty < x < \infty$). Show that the solution can also be put in the form $y(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)]$.
19. Use the convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$.
20. Apply the convolution theorem to prove that

$$B\,(\text{m, n}) = \int_0^1 u^{m-1}(1-u)^{n-1}\ du = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}, \text{ m}> 0 \text{ , n}> 0 \text{ .}$$

21. Define the term convolution on Fourier transform.

22. Define the inversion formula for Fourier sine transform of the function f(x) . What happens if f(x) is continuous?
23. State the Fredholm alternative concerning on integral equation.
24. Define singular integral equation with an example .
25. What is wavelet transformation?
26. Write a short note on FBI fringerprint compression?
27. Solve (tD^2+(1-2t)D -2) y = 0 ,y(0) =1, y^1(0) = 2 , where D≡ $\frac{d}{dx}$.
28. Show that if a function **f(x)** defined on $(-\infty\,,\infty)$ and its Fourier transform F(ζ) are both real, then **f(x)** is even. Also show that if f(x) is real and its Fourier transform **F(ζ)** is purely imaginary, then **f(x)** is odd.
29. If the function f(t) has the period T > 0 then prove that
30. L**{f(t)}** = $\frac{1}{1-e^{-pT}}\int_0^T f(t)e^{-pt}dt$.
31. Discuss the solution procedure for solving the homogeneous Fredholm integral equation with seperable kernel.
32. Find the eigen value and corresponding eigen functions of the integral equation . y(x) = $\lambda\int_0^{2\pi}\sin x\cos t\; y(t)dt$.
33. If the Fourier transform of f(x) is $\frac{\alpha}{1+\alpha^2}$, α being the transform parameter , then find f(x).
34. Show that if ψ is wavelet and ϕ is bounded integrable function , then the convolution function $\psi * \phi$ is a wavelet.
35. Prove that Haar wavelet is one of the most fundamental example of the general wavelet theory.
36. Convert $y''(x) - 3y'(x) + 2y(x) = 4\sin x$ with initial condition y(0) =1, $y'(0) = 2$ into a volterra integral equation of the second kind . Conversely derive the original differential equation with initial conditions from the integral equation obtained.
37.
38. Solve the integral equation y(x) = f(x) +$\lambda\int_{-1}^{1}(xt + x^2t^2)\, y(t)dt$.
39. Evaluate L{$\int_0^t \frac{\sin u}{u}\, du$} by the help of initial value theorem.
40. State and prove Parseval's identity on Fourier transform. Use generalization of Parseval's relation to show that $\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{ab(a+b)}$, a, b > 0.

41. Using the Fourier sine transform, solve the PDE $\frac{\partial v}{\partial t} = k\,\frac{\partial^2 v}{\partial x^2}$ for x > 0,t > 0 , under the boundary conditions $v = v_0$ when x = 0 , t > 0 and the initial condition v = 0 , when t = 0 , x > 0 .

42. From an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} - \sin(x)\frac{dy}{dx} + e^x y = x$ with the conditions y(0)=1, $y'(0) = -1$ and find its kernel.

43. Find the Laplace transform of the triangular wave function f(t) which is defined as follows
f(t)= t if $0 \le t < c$ and f(t)=2c-t if $c \le t < 2c$ where f(t+2c)=f(t).

44. Find the exponential Fourier transform of f(t) = $1 - |t|$ if |t|<1 and f(t)=0 if
|t|>1

45.

46. If the Fourier transform of f(x) is $\frac{\alpha}{1+\alpha^2}$, α being the transform parameter, then find f(x).

47. Using Laplace transform method solve the simultaneous equations
$\frac{dx}{dt}$ - y=e^t, $\frac{dy}{dt}$ + x =sin t, Given that x(0)=1 and y(0)=0.

48. Solve the integral equation
$y(x) = f(x) + \lambda \int_{-1}^{1} (x+t)\, y(t)dt$. And find the eigen values.

49. Evaluate $L\left\{\int_0^t \frac{\sin u}{u} du\right\}$ by the help of initial value theorem.

50. State and prove Parseval's identity on Fourier transform. Use generalization of Parseval's relation to show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{ab(a+b)}$, a, b > 0.

51. Use the convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$

52. State and prove convolution theorem on Laplace transform.

53. Solve the integral $\varphi(x) = \int_0^x \frac{1}{(x-t)^\alpha} y(t)dt$ $0 < \alpha < 1$.

54.

55. Find the eigen values and the corresponding eigen functions of the integral equation $y(x) = \lambda \int_0^1 (2xt - 4x^2) y(t)dt$

56. Solve the equation y(t) = e^{-t} - $2\int_0^t \cos(t-x)\, y(x)\ dx$,by using Laplace transform.

57. Reduce the boundary value problem $\frac{d^2y}{dx^2} + \lambda xy = 1$, $0 \leq x \leq l$ with boundary condition $y(0) = 0, y(e) = 1$ to an integral equation and its kernel .

58. Show that if a function $f(x)$ defind on $(-\infty, \infty)$ and its Fourier transform $F(\zeta)$ are both real , then $f(x)$ is even . Also show that if $f(x)$ is real and its Fourier transform $F(\zeta)$ is purely imaginary , then $f(x)$ is odd.

59. Solve one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, subject to the conditions u(x,0)=f(x),u_t(x,0)=0.

60. Prove that $\int_0^\infty \frac{s^3 sinsx}{s^4+4}\ ds = \frac{\pi}{2} e^{-x} cosx, x > 0$.

61. If $L[f(t)] = \frac{k}{s(s^2+4)}$ and $\lim_{t\to\infty} f(t)$=1, then find the value of k.

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